

The Little Logic Book
Hardy, Ratzsch, Konyndyk De Young and Mellema
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Exercises for The Little Logic Book may be downloaded by the instructor as Word documents and then modified for distribution to students; or students may be instructed to download the exercises and then told which ones to answer. This is an exercise bank; it is not assumed that students will do all the exercises for any one chapter. Comments, questions or suggestions for Chapter Four of The Little Logic Book can be sent to logic@calvin.edu

Exercise Bank for Chapter Four:
Modal Logic
(Posted January, 2014)

1.0 Basics Concepts

Define or identify the following:

- 1.1 Modal logic
- 1.2 Possible world
- 1.3 S5
- 1.4 Epistemic logic
- 1.5 Deontic logic

2.0 Possible Worlds

Translate the following into possible world terms:

- 2.1 P is necessarily true
- 2.2 P is necessarily false
- 2.3 P is possibly true
- 2.4 P is possibly false
- 2.5 P is contingent
- 2.6 P is in fact true
- 2.7 P is in fact false

3.0 Symbols

Identify the symbols for

- 3.1 P is necessary
- 3.2 P is possible

4.0 Modal Claims

In modal logic, “necessary” means logically necessary, that is, true in all possible worlds. In everyday discourse, we use the word “necessary” in a much looser and broader sense. Consider the following claims. Which ones speak of necessity in the sense of a statement that must be true in all possible worlds as opposed to a statement that must be true in this world, or in some but not all possible worlds?

- 4.1 It is necessary to get your driver’s license before you drive.

- 4.2 “2 plus 2 equals 4” is necessarily true.
- 4.3 A bachelor is necessarily an adult unmarried male.
- 4.4 Water is necessary for human life.
- 4.5 Necessarily, a thing cannot both be and not be at the same time.

5.0 Modal Principles

Explain why the following modal principles are true:

- 5.1 If P is necessary, then P is true.
- 5.2 If P is true, then P is possible
- 5.3 If P is necessary, then it is impossible for P to be false
- 5.4 If P is possible, then P is not a necessary falsehood.

6.0 Modal Logic and Possible Worlds

6.1 Discuss the following claim. Do you think it is true or not? Explain your answer.

If P is necessarily true, then it is necessarily true that P is necessarily true
(i.e., $\Box P \rightarrow \Box \Box P$).

(You may find it helpful to translate that into possible world terms.)

6.2 The following argument is a very well known modal *fallacy*. Can you explain why it is fallacious? (Note: “No, I can’t explain it.” is not really an adequate answer.)

$\Box(P \rightarrow Q)$
P
Therefore,
 $\Box Q$

(Again, thinking about this argument in terms of possible worlds may be very helpful.)

6.3 Why do the concepts and principles of modal logic play such an important role in some parts of philosophy?

6.4 On a scale of 1 – 10, exactly how weird *is* your philosophy instructor? Do not hand in your answer to this question.

6.5 Thinking in terms of possible worlds, explain why the following formulas work in the reverse direction:

- 6.5.1 $\Box P \rightarrow \sim \Diamond \sim P$
- 6.5.2 $\Diamond P \rightarrow \sim \Box \sim P$

i.e., explain why the following are true:

- 6.5.3 $\sim \Diamond \sim P \rightarrow \Box P$
- 6.5.4 $\sim \Box \sim P \rightarrow \Diamond P$

6.6 Again thinking in terms of possible worlds, explain why the following formulas do not work in the reverse direction:

- (1) $\Box P \rightarrow P$
- (2) $P \rightarrow \Diamond P$

i.e., explain why the following formulas are *not* true:

- (1a) $P \rightarrow \Box P$
- (2a) $\Diamond P \rightarrow P$

6.7 Explain in terms of possible worlds why the following formula is true:

$$\Diamond \sim P \rightarrow \sim \Box P$$

6.8 Sherlock Holmes frequently said:

“When you have eliminated the impossible, whatever remains—no matter how improbable—must be the truth.”

Despite his use of modal terms—e.g., “impossible”—it is unlikely that he meant to be making a precise modal statement. But suppose someone did intend it that way. Explain why *as a modal statement* the above would be false. (Hint: think about the category of *contingent* statements.)

7.0 A Stretch

Boethius was a Roman Christian philosopher of the late 5th and early 6th centuries. He wrote a number of important books on logic, and produced a number of translations and commentaries on Greek philosophers. But he was also a statesman. Adopted by a well-placed Roman family, he rose to power in the western Roman world and soon become the right-hand man of the Emperor Theodoric. But his good fortune soon turned. Theodoric, feeling threatened by the powers of the eastern empire and his own senate, accused Boethius of treason and had him sentenced to death. In prison, awaiting execution, Boethius wrote *The Consolation of Philosophy*, one of the most influential books in western Europe during the Middle Ages and the Renaissance. In this work, he imagines he is visited by Philosophy, personified as a woman, who seeks to raise him above his troubles by reminding him of the rule of God and the nature of his soul.

In the last book of the work, Book V, the conversation turns to God’s foreknowledge of our actions. It seems that there is a conflict between God’s foreknowledge of all things and human freedom. If God knows what I will do tomorrow, then I must do exactly what God knows I will do. Otherwise God would not know it. (Remember that you can’t know what is false. You can believe it. You can believe it very strongly. But you can’t *know* it. If you know something, then it’s true.) If I must do what God knows I will do in the future, then I cannot do otherwise. Do I really have a free choice in the matter if I cannot choose otherwise? If God knows that I will snap at my roommate tomorrow, then I must snap at my roommate tomorrow. It seems, then, I am fated to do so. I must do so. It is, to use a modal term, *necessary* that I do so.

Philosophy helps Boethius out of this problem by making a distinction between two different kinds of necessity: simple necessity and conditional necessity. Using symbols, we can represent the difference this way: $\Box Q$ is simple necessity; $\Box(P \rightarrow Q)$ is conditional necessity. When we use the concept of simple necessity we say: necessarily Q is the case. When we use the concept of conditional necessity we say: Necessarily, if P is the case, then Q is the case. To illustrate, there is a big difference in saying:

Necessarily, paying tuition next semester will be easy.
and

Necessarily, if you are a billionaire, then paying tuition next semester will be easy.

In our example, then, we can make a distinction between these two claims:

Necessarily, I will snap at my roommate tomorrow.

and

Necessarily, if God knows that I will snap at my roommate tomorrow, then I will snap at my roommate tomorrow.

Philosophy claims that the necessity of my doing what God knows I will do is a conditional necessity, not a simple necessity. How does this help her reconcile the apparent tension between God's foreknowledge and human freedom?